Routing and wavelength assignment in WDM optical networks : exact resolution vs. random search based heuristics

Résolution exacte et résolution approchée du problème de routage et affectation de longueurs d'onde dans les réseaux optiques WDM

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Résumé

Le problème de routage et d'affectation de longueurs d'onde (RWA) dans les réseaux optiques à multiplexage en longueurs d'onde (réseaux WDM) a été abondamment étudié depuis deux décennies. La plupart des études considèrent un trafic incrémental et traitent les demandes séquentiellement selon leur ordre d'arrivée. L'ordre selon lequel les demandes sont traitées a un impact considérable sur la qualité de la solution obtenue par l'algorithme.

Dans une précédente étude, les auteurs de ce rapport ont proposé un algorithme de routage et d'affectation de longueurs d'onde séquentiel amélioré dans le cas où l'on considère une matrice de demandes. Cet algorithme a pour but de trouver rapidement un ordre de traitement des demandes plus favorable que l'ordre initial des demandes. Le présent rapport a pour objet d'évaluer les performances de cet algorithme en proposant une comparaison systématique des solutions obtenues avec l'algorithme séquentiel amélioré aux solutions optimales obtenues par un modèle de programmation linéaire. On constate que les solutions obtenues par l'algorithme séquentiel amélioré sont, pour les situations étudiées, très proches des solutions optimales. L'efficacité et les performances de l'algorithme séquentiel amélioré justifient donc son utilisation.

Mots-clés : routage et affectation de longueurs d'onde, approche exacte, approche heuristique, réseaux à multiplexage en longueurs d'onde, WDM, RWA.

Routing and Wavelength Assignment in WDM Optical Networks: Exact Resolution vs. Random Search Based Heuristics

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Abstract

The routing and wavelength assignment (RWA) problem has been widely studied for the past decades in the framework of wavelength division multiplexing (WDM) optical networks. Most of the papers deal with incremental traffic and propose algorithms that serve the traffic demands sequentially according to their arrival order. Actually, the order according to which the demands are dealt with has a major impact on the quality of the RWA solution obtained by any sequential algorithm. In a preceding study, the authors of this report have designed an improved sequential RWA algorithm to deal with traffic matrices. The algorithm tries to find quickly an order that will lead to better results than the initial one. This paper aims to benchmark this new algorithm w.r.t. an ILP model that enables the user to compute the best solutions to the problem. The simulations show that the solutions obtained by the improved sequential algorithm are close to the optimal ones for the studied cases. Hence, the efficiency and the performance of the improved sequential RWA algorithm corroborate using it for RWA purposes, especially in situations when the problem size makes ILP model solving intractable.

1 Introduction

The aim of this paper is to prove the efficiency of the random search based heuristics (denoted RS) proposed in [1] to solve the routing and wavelength assignment problem (RWA) in WDM optical networks.

A network topology (fiber and nodes represented by a directed graph, weighted or not) and a demand matrix to be carried over the network (number of connection requests – demands – for any ordered pair of vertices) are given. We assume that all demands are known in advance and permanent (static traffic). The WDM technology allows a single fiber to carry a given number W of wavelengths, such that a wavelength can be used at most once on a given link. The objective of RWA is to maximize the number of established connections. A lightpath is used to support a connection, and must use the same wavelength on all the fiber links which it traverses (this property is known as the wavelength continuity constraint).

In order to demonstrate the efficiency of the RS algorithm designed in [1] to solve the static RWA problem, we compare it to the exact resolution of the problem.

This paper is organized as follows. In the second and third sections respectively, we present the integer linear programming formulation used for the exact resolution and the RS heuristics. In Section 4, we show that the search space may be reduced by considering only the five shortest paths. We justify this restriction by tests on random instances. Then, in Section 5, we analyse the behavior of both methods on random instances, and state through these simulations that the RS heuristics leads to rejection ratios close to the ones achieved by the exact model. Finally we conclude in Section 6.

2 Description of the exact model

The static RWA problem can be formulated as an integer linear program (ILP). Before presenting the exact model (designed in [3], [6]), we describe the parameters and variables of the ILP.

The parameters:

D denotes the number of active source-destination pairs (that is the number of node pairs that support at least one demand), these pairs are arbitrarily indexed from 1 to D ;

L is the number of links (directed edges in the graph);

W is the number of available wavelengths for any link;

P is the set of directed paths on which a connection may be routed (see Section 4);

$q = (q_i)$, $i \in \{1, ..., D\}$ is a line-vector in which q_i denotes the number of connections to be set up for the i-th source-destination pair;

$A = (a_{ij})$ is a binary $P \times D$ matrix in which $a_{ij} = 1$ if the i-th path joins the j-th source-destination pair and 0 otherwise;

$B = (b_{ij})$ is a binary $P \times L$ matrix in which $b_{ij} = 1$ if the j-th link belongs to the i-th path, and 0 otherwise.

The variables:

$m = (m_i)$, $i \in \{1, ..., D\}$ is a line-vector in which m_i denotes the number of connections established for the i-th source-destination pair;

$C = (c_{ij}) : P \times W$ is the route and wavelength assignment matrix, in which $c_{ij} = 1$ if a connection has been established using the i-th path and the j-th wavelength, and 0 otherwise.

According to these notations, the problem can be formulated as an ILP as follows :

$$\text{Maximize} \sum_{i=1}^{D} m_i$$

subject to :

$$C^T B \leq \mathbb{1}_{W \times L} \tag{1}$$

$$m = \mathbb{1}_W C^T A \tag{2}$$

$$m_i \leq q_i, i \in \{1, 2, ..., D\} \tag{3}$$

and with:

$$m_i \in \mathbb{N}, i \in \{1, 2, ..., D\} \tag{4}$$

$$c_{ij} \in \{0, 1\}, i \in \{1, 2, ..., P\}, j \in \{1, 2, ..., W\} \tag{5}$$

where $\mathbb{1}_{W \times L}$ (resp. $\mathbb{1}_W$) is the $W \times L$ (resp. $1 \times W$) matrix in which all the elements are 1.

Equation (1) specifies that a wavelength can be used at most once on a given link. Equation (2) defines m according to the route and wavelength assignment matrix C. Constraint (3) ensures that the number of established connections is lower than the number of requested connections. Constraints (4) and (5) correspond to domain constraints.

2.1 Symmetrical demands

In the previous formulation, we have considered directed source-destination pairs. Let us assume now that all demands are symmetrical (if α connections are requested from s to d, then α connections are also requested from d to s). We also assume that the established lightpaths must be symmetrical: any lightpath supporting a connection from s to d must correspond to a lightpath from d to s that follows the inverse path and with the same wavelength. These constraints impose that all the links used to carry some traffic have their symmetrical counterpart in the graph.

In order to exploit this symmetry, we consider the undirected graph associated to the network topology. Hence all the previous notions may be considered as undirected (edges, paths, source-destination pairs), and the same ILP formulation may be used to solve this new instance whose size is half the size of the initial one.

3 Description of the RS heuristics

Before explaining the principles of the RS-based algorithm, we describe the basic sequential RWA (`seqRWA`) algorithm. We assume that a set of candidate paths have been computed off-line for each source-destination pair (see Section 4). The `seqRWA` algorithm considers in turn the connection requests in an arbitrary order. For each path candidate to support the current connection request, we look for as many path-free wavelengths as the number of requested lightpaths. If the number of available wavelengths on a path $p \in P$ is higher than the number of requested lightpaths, wavelengths are assigned according to a First-Fit scheme [4]. The assigned wavelengths are reserved on p and on all the paths sharing a common link with p.

We notice that the solution given by `seqRWA` highly depends on the order in which the demands are examined. The objective of the random search is to find an order that minimizes the number of rejected demands obtained by the `seqRWA` algorithm.

Let Ω be the set of the $D!$ possible permutations of the set $\{1, 2, \ldots, D\}$. Each permutation, denoted $\rho_D^r, 1 \leq r \leq m \leq D!$, indicates an ordering of the lightpath demands to be routed. Let Ω' be a subset of Ω containing the permutations already investigated at the current step. The RS-based RWA algorithm is carried out as follows.

1. Select randomly an initial solution $\rho_D^1 \in \Omega$; $\Omega' := \{\rho_D^1\}$.
2. Repeat m times (for $r = 1tom$):
   (a) select randomly $\rho_D^r \in \{\Omega \setminus \Omega'\}$;
   (b) $\Omega' := \Omega' \cup \{\rho_D^r\}$;
   (c) $C_{\rho_D^r}$ is the number of rejected PLDs inherent to ρ_D^r;
3. Select in Ω' the vector $\rho_D^{r_0}, 1 \leq r_0 \leq m$ that minimizes the number of rejected lightpath demands.

   If two or more vectors in Ω' lead to the same minimum number of rejected PLDs, one selects the solution that minimizes the number of used WDM channels.

The number m of iterations is fixed according to the instance size or to the available resolution time.

4 Determination of the set P

In both the presented methods, we consider a set of paths P on which a connection may be routed. We restrict P to paths that have a pair $s - d$ as extremities such that there is at least one connection request from s to

network	number of vertices	number of edges	demand matrix	number of $s-d$ pairs	number of demands
$NSFnet14$	14	42	$M14$	70	100
$net29$	29	88	$M29$	70	200
$EBN57$	57	170	$M57$	71	355

Table 1: Networks and demand matrices characteristics.

d (the other paths are not useful for solving the problem). Moreover, the considered paths are elementary (i.e. cycle-free).

Now, in order to simplify the problem, we are not going to consider the set (which can be very large) of all elementary paths for all source-destination pairs, but only a limited number of the shortest elementary paths. We choose a number k and we compute, for any $s-d$ pair, the k shortest elementary paths from s to d (we use the method proposed by Yen [5], and Dijkstra's algorithm [2]).

The value of k must be chosen so that the quality of the solution is maintained and the time of resolution is moderate. To determine such a value, we test its influence over the exact resolution of the ILP presented in Section 2. We consider three networks extracted from the American optical network ($NSF14$ and $net29$) and the European network ($EBN57$). For each network, one demand matrix is generated randomly. The characteristics of the networks and of the demand matrices are given in Table 1.

We solved these three instances for different numbers of wavelengths ($W = 2, 5, 10, 15, 20$) and k varying from 1 to 20. The percentages of established connections are given in Figure 1.

For each case, we define $\bar{k}$ as the smallest integer value such that, $\forall k \in [\bar{k}, 20]$ the numbers of established connections are equal (i.e. it is not profitable to consider more than $\bar{k}$ shortest paths, as long as k remains lower than 20). The values of $\bar{k}$ are shown on each subfigure of Figure 1. We note that the number of considered shortest paths does not affect a lot the quality of the optimal solution since $\bar{k}$ remains relatively small in all cases. Of course, values of $\bar{k}$ are higher for larger networks.

The choice of $k = 5$ seems to be a good compromise since on the one hand it produces enough alternate paths to lead to a good solution and on the other hand it corresponds to reasonable data sizes (w.r.t. the network and matrix demand sizes). Table 2 gives the ratios of the value of the solution (number of established connections) obtained for $k = 5$ to $\bar{k}$. We notice that these ratios are always very high, close to 100%. It shows that the value $k = 5$, even for the cases when $\bar{k} \gg 5$, produces near optimal solutions (see for instance the network EBC57 with $W = 20$ whose $\bar{k}$ is equal to 19).

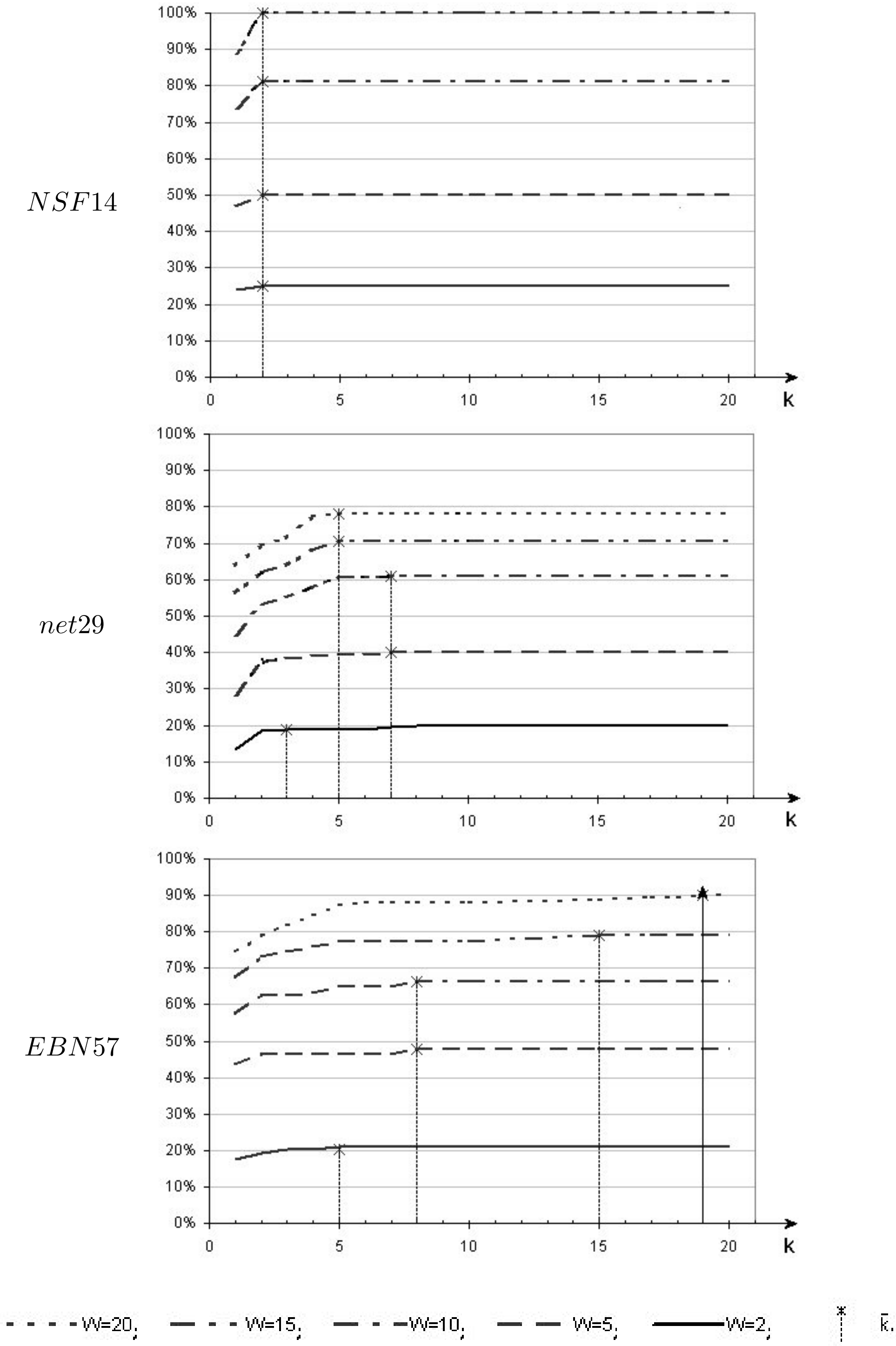

Figure 1: Percentages of established connections according to k for the three instances and different numbers of available wavelengths.

réseau	$W = 2$	$W = 5$	$W = 10$	$W = 15$	$W = 20$
$NSFnet14$	100%	100%	100%	100%	100%
$net29$	100%	98,75%	99,18%	100%	100%
$EBC57$	100%	97,06%	97,87%	98,21%	96,88%

Table 2: Percentages of established connections with $k = 5$ w.r.t. $\bar{k}$.

5 Comparison between RS and ILP

We now compare the results obtained with the RS heuristics with those given by the exact resolution of the ILP formulation (we used CPLEX 9.1.2.). We deal with the symmetrical version of the problem (symmetrical demands and lightpaths, see Section 2.1). We consider the $EBN57$ network (57 vertices, 170 edges) described previously (see Figure 2).

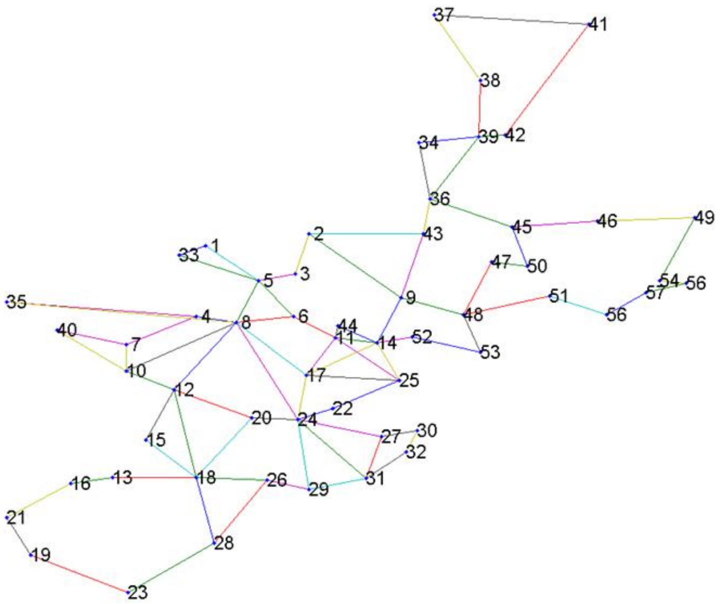

Figure 2: $EBN57$ european network topology.

We built randomly two groups A and B of demand matrices ($|A| = 3$, $|B| = 4$), whose characteristics are given in Table 3. In group A, the matrices include relatively few source-destination pairs and a variable number of demands. In group B, the matrices include roughly one demand by source-destination pair and the total number of demands involved is smaller than for matrices of group A. In other words, the traffic in the A group is rather concentrated on a few nodes whereas the traffic considered in the B group is scattered on the whole network.

demand matrix	A_1	A_2	A_3	B_1	B_2	B_3	B_4
number of symmetrical demands	296	355	1480	100	200	300	400
number of $s-d$ pairs	70	53	70	97	192	284	375

Table 3: Group A and group B demand matrices characteristics.

We solved the RWA problem for all these instances with both methods and for different values of W ($W = 10, 20, 30, 40$). We chose to examine 100 permutation vectors for the RS heuristic algorithm. The percentages of established connections for both the exact and the heuristic algorithm are given in Figure 3. The exact resolution (ILP) is represented in black, the RS heuristics in grey.

We have also compared the resolution times required by both methods; we noticed that this time does not depend strongly on the value of W. Figure 4 represents the averages of resolution times for each instance and for $W \in \{10, 20, 30, 40\}$. Exact resolution being very long for some instances, we stopped the resolution process prematurely. This was the case for $B3$ with $W = 10$, and for $B4$ with $W \in \{20, 30, 40\}$ whose solving were interrupted after several days of computation.

According to these results, we can draw several conclusions.

- The percentages of established connections increase with the number of available wavelengths and, of course, decrease when the number of requested connections increase.

- The greater the number of available wavelengths W is, the better the RS heuristics behaves. Indeed, for $W \in \{30, 40\}$, the percentages of established connections are close (or equal in most cases) to the optimal ones. For $W = 10$, the gap between ILP and RS is more important; it reaches 10% for $B2$, $B3$ and $B4$, which corresponds respectively to 19, 31 and 37 connections refused by the RS heuristics otherwise established by the exact method.

- With the RS heuristics, the resolution time remains small for any instance. However we can notice that times of resolution increase with the number of connection requests, reaching at best 20 seconds for $A1$ and $B1$ and at worst 5 minutes for $A3$.

- We remark that the exact resolution time mainly depends on the number of source-destination pairs D. The resolution is indeed very fast for instances of group A (even faster than the heuristics resolution times), but very long for instances of group B (more than several days).

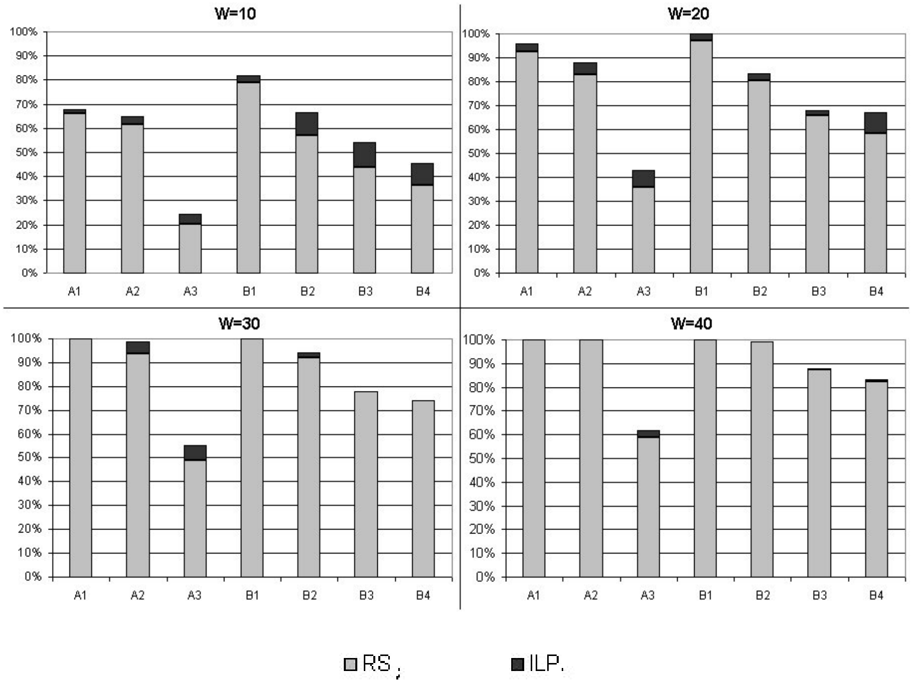

Figure 3: Percentages of established connections for the ILP and the RS methods applied on each problem instances on the $EBN57$ network.

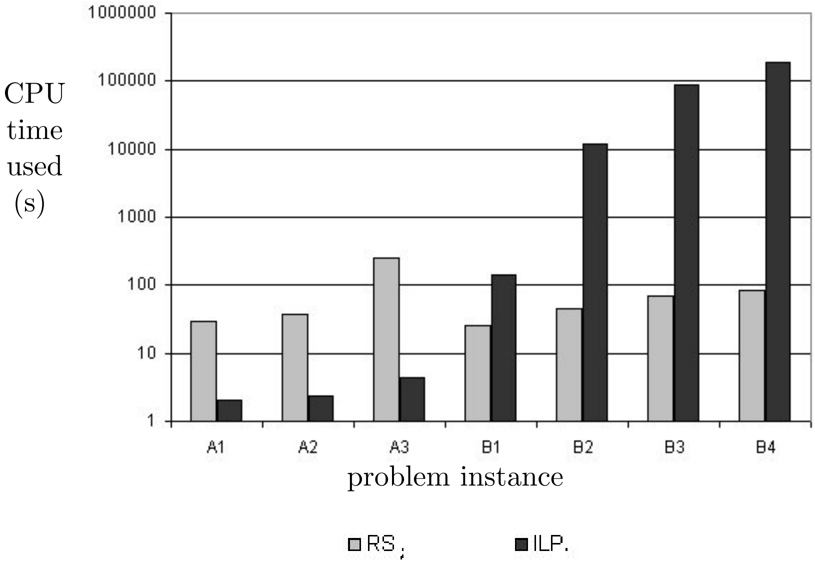

Figure 4: Average resolution times for the ILP and the RS methods applied on each problem instances on the $EBN57$ network in seconds.

Finally, we can conclude that the RS heuristics is efficient especially for great values of W. This can be explained by the fact that the choices made during the RS heuristics and which might lead to bad solutions, have less consequences when W is great. The exact resolution can be used for instances with a moderated number of source-destination pairs and few available wavelengths. Conversely, the RS heuristics should be used for large numbers of source-destination pairs and available wavelengths.

Another interesting point is to minimize the number of used WDM channels: more precisely, among all solutions maximizing the number of established connections, we would prefer the ones that minimize the number of used channels. The RS heuristics takes this aspect into account since the solution which minimizes the number of WDM channels is preferred when several vectors reject the same number of demands.

In order to compare the RS heuristics to the exact resolution according to this new criterion, we slightly changed the previous ILP formulation (see Section 2): we added a constraint that sets the number of established connections equal to a value θ (the number of connections established by the RS heuristics) and modified the objective function that is now to minimize the number of used WDM channels:

$$\text{Minimize} \sum_{i \in \{1,\ldots P\}, k \in \{1,\ldots W\}} \left(c_{ik} \sum_{j \in \{1,\ldots,L\}} b_{ij} \right)$$

We solved this problem for the 4 smallest demand matrices A_1,A_2, A_3 and B_1 with $W \in \{10, 20, 30, 40\}$. We denote by WDM_{ILP} the optimal objective value of such a problem and by WDM_{RS} the number of WDM channels required by the solution computed by the RS heuristics. Figure 5 represents the values:

$$\delta = \frac{\text{WDM}_{\text{RS}} - \text{WDM}_{\text{ILP}}}{\text{WDM}_{\text{ILP}}} \times 100.$$

We notice that WDM_{RS} decreases when W increases. It is near optimal for the instances A_1 with $W \in \{30, 40\}$, A_2 with $W = 40$ and B_1 with $W \in \{30, 40\}$. The behavior of RS is worse for the matrix A_3, that contains a lot of connection requests (1480), and for small values of W.

Again we notice the very satisfactory behavior of the RS heuristics when the number of available wavelength is large. Indeed, it leads to near optimal solutions in terms of number of established connections and in terms of number of used WDM channels.

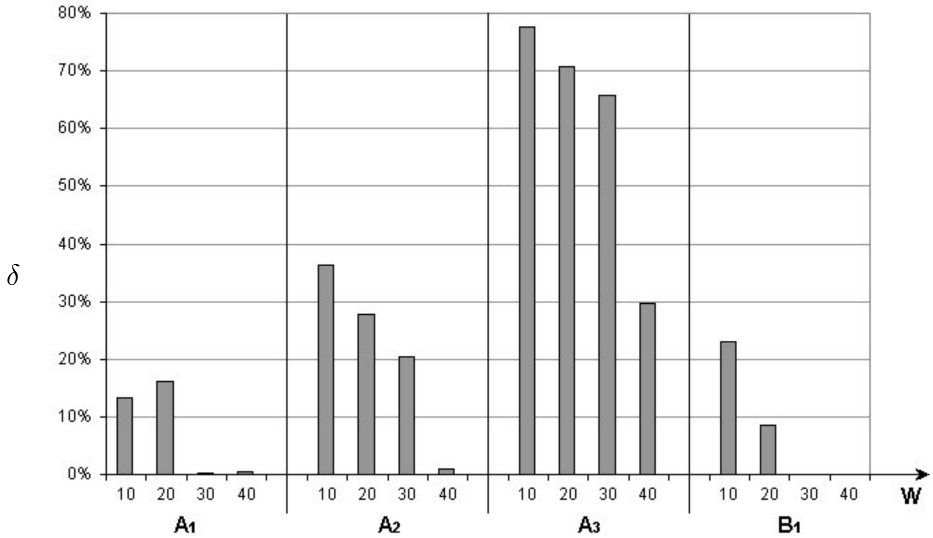

Figure 5: Gaps between $\mathrm{WDM}_{\mathrm{RS}}$ and $\mathrm{WDM}_{\mathrm{ILP}}$ for each instance.

6 Conclusion

In this paper, we analyzed a random search based heuristics (RS) designed to solve the static RWA problem. This problem can also be solved as an integer linear program (ILP) but the resolution of the exact model may be untractable for large size instances. In order to assess the use of the proposed RS heuristics, we applied it on several problem instances (of reasonable sizes) and compared the obtained results with the optimal ones.

According to this study, we conclude that the RS heuristics is efficient to solve RWA problem instances especially when the number of available wavelengths is great. Indeed, it gives near optimal solutions in short time. It is interesting to use this approximate method if the number of source-destination pairs is large, since, in this case, the exact model fails to find a solution in reasonable time. Moreover, we showed that solutions given by RS consume relatively few WDM channels, which confers another advantage to this heuristics.

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