Blind filter identification and image superresolution using subspace methods

Superrésolution aveugle d'images par la méthode des sous-espaces

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Résumé:

Les méthodes de sous-espaces sont un outil puissant d'identification aveugle des filtres par l'étude de statistiques du second ordre de plusieurs sorties issues d'une même source (problème dit SIMO). L'extension de ce problème au cas de sources multiples peut être envisagée, et a été développée dans la littérature.

Dans ce rapport, nous montrons comment ces méthodes permettent de résoudre le problème de superrésolution aveugle. Nous présentons tout d'abord le problème de superrésolution comme un problème à entrées multiples et à sorties multiples (MIMO). Nous montrons que la méthode de sous-espace ne peut être utilisée seule pour retrouver les filtres affectant chaque image, et nous proposons deux solutions possibles utilisant les propriétés statistiques des images pour résoudre le problème. Nous présentons des résultats expérimentaux qui valident notre approche.

Abstract :

Subspace methods are a powerful tool to recover unknown filters by looking at the second order statistics of various signals originating from the same source (also called a SIMO problem). An extension to the multiple source case is also possible and has been investigated in the literature.

In this report we show how the blind superresolution problem can be solved by this tool. We first present the problem of superresolution as a multiple input multiple output (MIMO) one. We show that the subspace method can not be used, as is, to recover the filters affecting each image, and we present two possible solutions, based on the statistical characteristics of the images to solve this problem. Experiments are shown which validate these ideas.

Contents

Introduction

The subspace method has been introduced by [7] and further investigate in a multitude of papers [1, 5, 6, 8]. The idea of the method is to observe multiple outputs of various unknown filters having all the same input (namely SIMO system). In this case, the second order statistics of the received signals carry enough information to allow the recovery of the filters and furthermore the recovery of the original signal.
The main application that authors had in mind was to conceive wireless protocols in a varying environment, in which no training sequence have to be transmitted. Indeed, in such an environment, the filters that affect the signal can change and have to be re-learned very often. Being able to learn them without the use of a training signal would be a great asset and could save an important amount of bandwidth.
Extensions to the case where a multitude of signals are transmitted through the same channel (namely MIMO systems) have been investigated [1, 5]. In this late approach, a crucial step is the use of a source separation technique.

We investigate the possibility of using the subspace method in the context of image superresolution. More precisely, we observe a known number of images of the same scene acquired through various filters and subsampled (the subsampling accounts for the aliasing that occurs in every image acquisition process). We would like to recover the original image and do so in two steps: the first step is to recover the filters using the subspace technique, the second step is to apply a regularized inversion to the observed images in order to recover the original scene.

This report is divided as follows:
Chapter 1 presents the subspace method for 1D signal and images, in order to provide the reader with a self contained overview.
Chapter 2 states the problem of superresolution as a multiple input multiple output (MIMO) one, in which the multiple inputs are the various subsampled versions of the image (they differ by a translation). This presentation allows us to understand that:

- The separation of sources is impossible in the case of superresolution because the sources are very correlated with each other and have exactly the same statistics (Section 2.2).
- The subspace method provides us with a mixture of the actual filters. Therefore, we have to implement a method to unmix and recover the actual filters. In the same time, the subspace method allows us to restrain the search for the filters to a relatively small affine space. In Section 2.3, we introduce our method to disambiguate the results of the subspace method and recover the actual filters.

In Chapter 3, we provide the restoration step based on the minimization of a functional

including a data driven term and a regularization one.
Finally, Chapter 4 presents experimental results for both filters recovery and signal/image restoration based on this recovery.

Chapter 1

The subspace method

In this chapter, we present the subspace method such as developed by [6] for 1D signals. This method, first introduced by [7], considers multi output systems. This allows the use of second order statistics of the outputs, instead of higher order statistics, to identify blindly the filters. This method, under some mild assumptions, estimates the noise and signal subspaces from the eigenvalue decomposition of the autocorrelation matrix of the outputs, and exploits the orthogonality between this subspaces to identify the filter coefficients.
First, we state the problem for 1D signals and extend the formulation to images, then, we develop the subspace method for images. We refer the reader to [6] for a presentation of the subspace method for 1D signals.

1.1 Problem statement

1.1.1 1D signals

Let us consider L signals X^l as noisy outputs of an unknown system driven by an unknown input D. We aimed to identify the system function H blindly, i.e. using only the outputs of the system.
The outputs are described by a convolution model:

$$X^l(.) = H^l(.) * D(.) + B(.) \tag{1.1}$$

where $*$ denotes the convolution, and B is a white zero-mean noise.

Using a matrix formulation, we can write :

$$X = \mathcal{H}D + B \tag{1.2}$$

where

- X stacks the L output signals of size $(N, 1)$:

$$X = \begin{pmatrix} X^1 \\ \vdots \\ X^L \end{pmatrix} = \begin{pmatrix} x_0^1 & \dots & x_{N-1}^1 & \dots & x_0^L & \dots & x_{N-1}^L \end{pmatrix}^T \tag{1.3}$$

- $\mathcal{H}$ stacks the L Toeplitz matrices $\mathcal{H}^l$:

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}^1 \\ \vdots \\ \mathcal{H}^L \end{pmatrix} \qquad (LN, N+M-1) \tag{1.4}$$

where $\mathcal{H}^l$, for $l = 1$ to L, is the filtering matrix associated to the filter H^l of size $(M, 1)$:

$$H^l = [h_0^l \dots h_{M-1}^l]^T \tag{1.5}$$

i.e. :

$$\mathcal{H}^l = \begin{pmatrix} h_0^l & \dots & h_{M-1}^l & & 0 \\ & \ddots & & \ddots & \\ 0 & & h_0^l & \dots & h_{M-1}^l \end{pmatrix} \qquad (N, N+M-1) \tag{1.6}$$

- D is the unknown original signal of size $(N + M - 1, 1)$,
- and B is a white zero-mean noise, assumed to be uncorrelated with D.

1.1.2 Images

Let us consider L blurred images acquired by multiple cameras, or by a single camera through different conditions (focus changes, camera motion...). The L observed images are modeled as noisy outputs of a FIR system $\mathcal{H}$ driven by an input original image D :

$$X = \mathcal{H}D + B \tag{1.7}$$

where :

- X stacks the L observed images X^l, for $l = 1$ to L, more precisely, a vectorized formulation of a processing windowed area, of size $(N_y\ ,\ N_x)$, extracted from the observed images :

$$X^l = [x^l(N_y-1, N_x-1)\ x^l(N_y-2, N_x-1)\ \cdots\ x^l(0,0)]^T \tag{1.8}$$

- D is a vectorized formulation of the related windowed area of the original image of size $(N_y + M_y - 1, N_x - M_x - 1)$,

$$D = [d(N_y + M_y - 2, N_x + M_x - 2) \cdots\ d(0,0)]^T \tag{1.9}$$

- $\mathcal{H}$ stacks the L block-Toeplitz filtering matrices $\mathcal{H}^l$ associated with each filters H^l of size (M_y, M_x)

$$H^l = \begin{pmatrix} h^l(0,0) & \dots & h^l(0, M_x-1) \\ \vdots & & \vdots \\ h^l(M_y-1, 0) & \dots & h^l(M_y-1, M_x-1) \end{pmatrix} \tag{1.10}$$

i.e. :

$$\mathcal{H}^l = \begin{pmatrix} \mathcal{H}_0^l & \cdots & \mathcal{H}_{M_x-1}^l & & 0 \\ & \ddots & & \ddots & \\ 0 & & \mathcal{H}_0^l & \cdots & \mathcal{H}_{M_x-1}^l \end{pmatrix} \tag{1.11}$$

where $\mathcal{H}_j^l$ is a Toeplitz matrix of size $(N_y\ ,\ N_y + M_y - 1)$ associated to the j^{th} column of H^l :

$$\mathcal{H}_j^l = \begin{pmatrix} h^l(0,j) & \dots & h^l(M_y-1,j) & & 0 \\ & \ddots & & \ddots & \\ 0 & & h^l(0,j) & \dots & h^l(M_y-1,j) \end{pmatrix} \tag{1.12}$$

$\mathcal{H}^l$ contains N_x rows of blocks and $N_x + M_x - 1$ columns of blocks of size $(N_y\ ,\ N_y + M_y - 1)$. $\mathcal{H}$ is of size $(LN_yN_x\ ,\ (N_y + M_y - 1)(N_x + M_x - 1))$,

- and B is a white zero-mean noise, assumed to be uncorrelated with D.

1.2 The subspace method for SIMO systems

Let $\mathbb{R}_X$ denotes the autocorrelation matrix of the outputs X:

$$\mathbb{R}_X = E(XX^T) \tag{1.13}$$

where E denotes the expectation operator. $\mathbb{R}_X$ is of size $(LN_xN_y\ ,\ LN_xN_y)$.

From equation (1.2) we deduce that:

$$\mathbb{R}_X = \mathcal{H}\mathbb{R}_D\mathcal{H}^T + \mathbb{R}_B \tag{1.14}$$

where $\mathbb{R}_D$ and $\mathbb{R}_B$ denote respectively the autocorrelation matrices of the input D and the noise B. We recall that the noise is assumed to be uncorrelated with the input.

From now on, we make two assumptions:

1. $\mathcal{H}$ is full column rank, a necessary condition is :

$$LN_yN_x > (N_x + M_x - 1)(N_y + M_y - 1) \tag{1.15}$$

2. and $\mathbb{R}_D$ is full rank.

We deduce from eq. (1.14) and thanks to these assumptions, that the signal part of the autocorrelation matrix $\mathbb{R}_X$, i.e. $\mathcal{H}\mathbb{R}_D\mathcal{H}^T$, has rank

$$d_H = (N_x + M_x - 1)(N_y + M_y - 1) \tag{1.16}$$

Through an eigenvalue decomposition of $\mathbb{R}_X$, we obtain a subspace decomposition between the signal and noise subspaces:

- The signal subspace is spanned by the eigenvectors associated with the d_H largest eigenvalues of $\mathbb{R}_X$
- The noise subspace, its orthogonal complement, is spanned by the eigenvectors associated with the $LN_xN_y - d_H$ smallest eigenvalues of $\mathbb{R}_X$.

The signal subspace is also the subspace spanned by the columns of the filtering matrix $\mathcal{H}$.

By orthogonality between signal and noise subspaces, we deduce that each vector of the noise subspace is orthogonal to each column of the filtering matrix.
Let G_i denotes an eigenvector associated with one of the $LN_xN_y - d_H$ smallest eigenvalues of the matrix $\mathbb{R}_X$. The orthogonality condition can be formulated, for $i = 0 : LN_xN_y - d_H - 1$, as:

$$\underset{(1,LN_yN_x)(LN_yN_x,d_H)}{G_i^T \mathcal{H}} = \mathbf{0}_{(1,d_H)} \tag{1.17}$$

where $\mathbf{0}_{(1,d_H)}$ is a null vector of size $(1, d_H)$.

Since we have only an estimate of the autocorrelation matrix, the orthogonality condition is solved using a least square method. This leads to the minimization of the quadratic form:

$$q(\mathcal{H}) = \sum_{i=0}^{LN_xN_y-d_H-1} |G_i^T \mathcal{H}|^2 \tag{1.18}$$

Thanks to the following structural lemma, we provide an expression of the quadratic form in terms of the filter coefficients instead of the filtering matrix :

Lemma 1 :

$$G_i^T \mathcal{H} = H^T \mathcal{G}_i \tag{1.19}$$

You can find a proof of this lemma in [6].

In this expression, the matrix $\mathcal{G}_i$, for $i = 0 : LN_xN_y - d_H - 1$, denotes a matrix of size $(LM_yM_x\ ,\ d_H)$. This matrix is constructed as follows:

- Each eigenvector G_i, for $i = 0$ to $LN_xN_y - d_H - 1$ is partitioned into L vectors G_i^l of size $(N_yN_x\ ,\ 1)$.

- Each part G_i^l can be considered as a vectorized formulation of the matrix:

$$G_i^l = \begin{pmatrix} g_i^l(0,0) & \dots & g_i^l(0, N_x-1) \\ \vdots & & \vdots \\ g_i^l(N_y-1, 0) & \dots & g_i^l(N_y-1, N_x-1) \end{pmatrix} \tag{1.20}$$

  Note that we use the same notation for both the matrix form or the vectorized form of G_i^l, as the reader can easily differentiate them by looking at the size of G_i^l in the given expression.

- Let us define the block-Toeplitz matrix $\mathcal{G}_i^l$ as the "filtering" matrix associated to G_i^l. The term "filtering" points out that we obtain $\mathcal{G}_i^l$ from G_i^l in the same way we obtain $\mathcal{H}^l$ from H^l (see eq. (1.11) and (1.12)).

$$\mathcal{G}_i^l = \begin{pmatrix} \mathcal{G}_{i,0}^l & \cdots & \mathcal{G}_{i,N_x-1}^l & & 0 \\ & \ddots & & \ddots & \\ 0 & & \mathcal{G}_{i,0}^l & \cdots & \mathcal{G}_{i,N_x-1}^l \end{pmatrix} \tag{1.21}$$

where $\mathcal{G}_{i,j}^l$ is a Toeplitz matrix of size $(M_y\ ,\ M_y + N_y - 1)$ associated to the j^{th} column of G_i^l in the matrix form (see eq. (1.20)):

$$\mathcal{G}_{i,j}^l = \begin{pmatrix} g_i^l(0,j) & \dots & g_i^l(N_y-1,j) & & 0 \\ \ddots & & & \ddots & \\ 0 & g_i^l(0,j) & & \dots & g_i^l(N_y-1,j) \end{pmatrix} \tag{1.22}$$

$\mathcal{G}_i^l$ contains M_x rows of blocks and $M_x + N_x - 1$ columns of blocks of size $(M_y\ ,\ My + N_y - 1)$.

- Finally, $\mathcal{G}_i$ stacks the L matrices $\mathcal{G}_i^l$ and is of size (LM_yM_x, d_H).

The quadratic form is now expressed in terms of the filter coefficients:

$$q(H) = H^T \mathbb{Q} H \ \text{ where } \ \mathbb{Q} = \sum_{i=0}^{LN_xN_y - d_H - 1} \mathcal{G}_i \mathcal{G}_i^T \tag{1.23}$$

The filter coefficients are identified, up to a constant, by the minimal eigenvector of $\mathbb{Q}$.

Chapter 2

Extension to subsampled signals

We now extend the subspace-based method to the case of subsampled observed images. The subsampling accounts for the aliasing that occurs in every image acquisition process. The purpose is to estimate, from the low-resolution observed images, a deconvolved image at a higher resolution: this problem is called superresolution. To this end, we assume that the original image is filtered by L high-resolution filters, and the L output images are then subsampled by a factor P. The estimation is blind, i.e. we do not know the filters.
In this chapter, we first state the problem of superresolution as a MIMO one, for 1D signals and for images (section 2.1). Then, in section 2.2, we focus on the limits of the subspace method for MIMO systems. For MIMO systems, the subspace method provide only a mixture of the filters, and no more the actual filters, such as in the SIMO case. Source separation methods have been used to unmix the result of the subspace method and retrieve the actual filters, but these methods assume that the input signal are not correlated. In our case, the inputs are strongly correlated as they are the various subsampled version of the same image. We present in section 2.3 our method to disambiguate the mixture, and provide the actual filter, for subsampled input signals.

2.1 Problem statement

2.1.1 1D signals

Each observed signal X^l is modeled as a noisy output of a FIR system driven by an input D (see eq. (1.2) page 8) :

$$\underset{(N,1)}{X^l} = \underset{(N,N+M-1)}{\mathcal{H}^l} \underset{(N+M-1,1)}{D} + \underset{(N,1)}{B^l} \tag{2.1}$$

where $\mathcal{H}^l$ is the filtering matrix associated to the filter H^l, for $l = 1$ to L.

After the convolution step, the output signals are subsampled by a factor P. These subsampled signals, denoted X^l_{LR}, can be deduced from D following :

$$\underset{(n,1)}{X^l_{LR}} = \underset{(n,P(n+m-1))}{\mathcal{H}^l_{LR}} \underset{(P(n+m-1),1)}{D} + \underset{(n,1)}{B^l_{LR}} \tag{2.2}$$

where $n = \frac{N}{P}$, $m = \frac{M}{P}$ and D is the original signal cut by its $P-1$ last samples. Note that we assume that N and M are multiples of P.

We obtain the filtering matrix $\mathcal{H}^l_{LR}$ by extracting one row every P from the filtering matrix $\mathcal{H}^l$. If $\mathcal{H}^l$ is written by :

$$\mathcal{H}^l = \begin{pmatrix} h^l_0 & \dots & h^l_{M-1} & 0 & \dots & \dots & 0 \\ 0 & h^l_0 & \dots & h^l_{M-1} & 0 & \dots & 0 \\ \vdots & & \ddots & & \ddots & & \vdots \\ 0 & \dots & 0 & h^l_0 & \dots & h^l_{M-1} & 0 \\ 0 & \dots & \dots & 0 & h^l_0 & \dots & h^l_{M-1} \end{pmatrix} \qquad (N, M+N-1) \tag{2.3}$$

the filtering matrix $\mathcal{H}^l_{LR}$ is written by :

$$\mathcal{H}^l_{LR} = \begin{pmatrix} h^l_0 & \dots & h^l_{M-1} & 0 & \dots & \dots & 0 \\ \underbrace{0\dots0}_{P \text{ zeros}} & h^l_0 & \dots & h^l_{M-1} & 0 & \dots & 0 \\ \vdots & & \ddots & & \ddots & & \vdots \\ 0 & \dots & 0 & h^l_0 & \dots & h^l_{M-1} & \underbrace{0\dots0}_{P \text{ zeros}} \\ 0 & \dots & \dots & 0 & h^l_0 & \dots & h^l_{M-1} \end{pmatrix} \qquad (n, P(m+n-1)) \tag{2.4}$$

Let us denote H^l_p the subsampled component of the filter H^l, we also name it a polyphase component of H^l :

$$H^l_p = \left[\, h_p \; h_{p+P} \; \dots \; h_{p+(m-1)P} \,\right] \tag{2.5}$$

The filtering matrix associated to H^l_p is denoted by $\mathcal{H}^l_p$ and is of the form :

$$\mathcal{H}^l_p = \begin{pmatrix} h^l_p & h^l_{p+P} & \dots & h^l_{p+(m-1)P} & & 0 \\ \ddots & & & & \ddots & \\ 0 & & h^l_p & h^l_{p+P} & \dots & h^l_{p+(m-1)P} \end{pmatrix} \qquad (n, n+m-1) \tag{2.6}$$

By switching on purpose the columns of $\mathcal{H}^l_{LR}$, we obtain an expression of $\mathcal{H}^l_{LR}$ in terms of the filtering matrices associated to the polyphase components of the filter H^l :

$$\mathcal{H}^l_{LR} = \begin{pmatrix} \underbrace{\begin{matrix} h^l_0 & h^l_P & \dots & h^l_{(m-1)P} & 0 \\ \ddots & & & \ddots & \\ 0 & h^l_0 & h^l_P & \dots & h^l_{(m-1)P} \end{matrix}}_{\mathcal{H}^l_0} & \dots & \underbrace{\begin{matrix} h^l_{P-1} & h^l_{2P-1} & \dots & h^l_{mP-1} & 0 \\ \ddots & & & \ddots & \\ 0 & h^l_{P-1} & h^l_{2P-1} & \dots & h^l_{mP-1} \end{matrix}}_{\mathcal{H}^l_{P-1}} \end{pmatrix} \tag{2.7}$$

By switching at the same time the relating rows of D, we obtain an expression of D in terms of its subsampled component :

$$D = \begin{pmatrix} D_0 \\ \vdots \\ D_{P-1} \end{pmatrix} \tag{2.8}$$

where D_p, for $p = 0$ to $P-1$, denotes a subsampled component of D :

$$D_p = [d_p \; d_{p+P} \dots d_{p+(n+m-2)P}]^T \qquad (n+m-1, 1) \tag{2.9}$$

Equation (2.2) can be written as :

$$\begin{array}{ccccccc} X_{LR}^{l} & = & \begin{pmatrix}\mathcal{H}_0^l & \dots & \mathcal{H}_{P-1}^l\end{pmatrix} & \begin{pmatrix} D_0 \\ \vdots \\ D_{P-1}\end{pmatrix} & + & B_{LR}^{l} \\ (n,1) & & (n,P(n+m-1)) & (P(n+m-1),1) & & (n,1) \end{array} \tag{2.10}$$

Let us illustrate this step by an example :

$D = [d_0\ d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7\ d_8\ d_9\ d_{10}]^T$ denotes a signal of size $M + N - 1 = 11$, filtered by $H^l = [h_0\ h_1\ h_2\ h_3\ h_4\ h_5]$ of size $M = 6$, and noisy.
The resulting observed signal, denoted by $X^l = [x_0\ x_1\ x_2\ x_3\ x_4\ x_5]$, is of size $N = 6$.
The output signal X^l is related to D following :

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \\ d_9 \\ d_{10} \end{pmatrix} + B^l \tag{2.11}$$

Then the output signal is subsampled by a factor $P = 3$ and the subsampled output signal X_{LR}^l is expressed by :

$$\begin{pmatrix} x_0 \\ x_3 \end{pmatrix} = \begin{pmatrix} h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \end{pmatrix} + B_l^{LR} \tag{2.12}$$

By switching the columns of the filtering matrix, and at the same time the rows of the original signal, we obtain :

$$\begin{pmatrix} x_0 \\ x_3 \end{pmatrix} = \begin{pmatrix} h_0 & h_3 & 0 & h_1 & h_4 & 0 & h_2 & h_5 & 0 \\ 0 & h_0 & h_3 & 0 & h_1 & h_4 & 0 & h_2 & h_5 \end{pmatrix} \begin{pmatrix} d_0 \\ d_3 \\ d_6 \\ d_1 \\ d_4 \\ d_7 \\ d_2 \\ d_5 \\ d_8 \end{pmatrix} + B_l^{LR} \tag{2.13}$$

The filtering matrix can be partitioned into three sub-matrices :

$$\mathcal{H}_0^l = \begin{pmatrix} h_0 & h_3 & 0 \\ 0 & h_0 & h_3 \end{pmatrix} \quad \mathcal{H}_1^l = \begin{pmatrix} h_1 & h_4 & 0 \\ 0 & h_1 & h_4 \end{pmatrix} \quad \mathcal{H}_2^l = \begin{pmatrix} h_2 & h_5 & 0 \\ 0 & h_2 & h_5 \end{pmatrix} \tag{2.14}$$

which can be seen as the filtering matrices of the polyphase components $H_0^l = [h_0\ h_3]^T$, $H_1^l = [h_1\ h_4]^T$, and $H_2^l = [h_2\ h_5]^T$.
Let us denotes $D_0 = [d_0\ d_3\ d_6]$, $D_1 = [d_1\ d_4\ d_7]$ and $D_2 = [d_2\ d_5\ d_8]$. D_p, for $p = 0$ to $P - 1$, is a subsampled components of the original signal.

Subsampled output signal X_{LR}^l can be expressed in terms of the subsampled components of the original signal D :

$$X_l^{LR} = \begin{pmatrix} \mathcal{H}_0^l & \mathcal{H}_1^l & \mathcal{H}_2^l \end{pmatrix} \begin{pmatrix} D_0 \\ D_1 \\ D_2 \end{pmatrix} + B^{LR} \tag{2.15}$$

Through this example, and in a more general case, we show that the superresolution problem can be stated as a multiple input multiple output (MIMO) system, more precisely a P input and L output system, where the inputs are the subsampled components of the original signal.

$$\underset{(Ln,1)}{\begin{pmatrix} X_{LR}^1 \\ \vdots \\ X_{LR}^L \end{pmatrix}} = \underset{(Ln,P(n+m-1))}{\begin{pmatrix} \mathcal{H}_0^1 & \dots & \mathcal{H}_{P-1}^1 \\ \vdots & & \vdots \\ \mathcal{H}_0^L & \dots & \mathcal{H}_{P-1}^L \end{pmatrix}} \underset{(P(n+m-1),1)}{\begin{pmatrix} D_0 \\ \vdots \\ D_{P-1} \end{pmatrix}} + \underset{(Ln,1)}{B_{LR}} \tag{2.16}$$

2.1.2 Images

Each observed image X^l, $l = 1 : L$, is modeled as a noisy output of a FIR system $\mathcal{H}^l$ driven by an input image D (see section 1.1.2):

$$X^l = \mathcal{H}^l D + B^l \tag{2.17}$$

Then, the outputs are subsampled by a factor P:

$$X_{LR}^l = \mathcal{H}_{LR}^l D + B_{LR}^l \tag{2.18}$$

In this expression :

- X_{LR}^l is a subsampled component of X^l, of size $(n_x n_y, 1)$,
  where $n_x = \frac{N_x}{P}$ and $n_y = \frac{N_y}{P}$,
- D is the same as in equation (2.17), apart from the last $P - 1$ rows and columns which are truncated,
- $\mathcal{H}_{LR}^l$ is defined by extracting one row every P from the matrix $\mathcal{H}^l$
  and is of size $(n_x n_y, d_h)$, as we discard all the null columns,
  where $d_h = P^2(n_x + m_x - 1)(n_y + m_y - 1)$, $m_x = \frac{M_x}{P}$ and $m_y = \frac{M_y}{P}$.

By switching on purpose the columns of $\mathcal{H}_{LR}^l$ (and at the same time the rows of D) in equation (2.18), the subsampled output images can be related to the subsampled components of the original image:

$$X_{LR}^l = \begin{pmatrix}\mathcal{H}_{0,0}^l & \mathcal{H}_{0,1}^l & \dots & \mathcal{H}_{P-1,P-1}^l\end{pmatrix}\begin{pmatrix}D_{0,0}\\ D_{0,1}\\ \vdots \\ D_{P-1,P-1}\end{pmatrix} + B_{LR}^l \tag{2.19}$$

where :

- D_{p_1,p_2} is a vectorized subsampled component of the input image D, i.e., if

$$D = \begin{pmatrix} d_{0,0} & \dots & d_{0,S_x-1}\\ \vdots & & \vdots \\ d_{S_y-1,0} & \dots & d_{S_y-1,S_x-1}\end{pmatrix} \tag{2.20}$$

  where $S_y = N_y + M_y - 1$ and $S_x = N_x + M_x - 1$,
  thus, for all $p_1, p_2 = 0 : P-1$, then D_{p_1,p_2} is a vectorized form of

$$D_{p_1,p_2} = \begin{pmatrix} d_{p_1,p_2} & d_{p_1,p_2+P} & \dots & d_{p_1,p_2+P(s_x-1)}\\ \vdots & \vdots & & \vdots \\ d_{p_1+P(s_y-1),0} & d_{p_1+P(s_y-1),P} & \dots & d_{p_1+P(s_y-1),p_2+P(s_x-1)}\end{pmatrix} \tag{2.21}$$

  where $s_y = n_y + m_y - 1$ and $s_x = n_x + m_x - 1$,
  note that we use the same notation for the vectorized and the matrix forms of D_{p_1,p_2},

- and $\mathcal{H}_{p_1,p_2}^l$ is the block-Toeplitz matrix of size $(n_y n_x, s_y s_x)$ associated to the filter

$$H_{p_1,p_2}^l = \begin{pmatrix} h_{p_1,p_2}^l & \dots & h_{p_1,p_2+(m_x-1)P}^l\\ h_{p_1+P,p_2}^l & \dots & h_{p_1+P,p_2+(m_x-1)P}^l\\ \vdots & & \vdots \\ h_{p_1+(m_y-1)P,p_2}^l & \dots & h_{p_1+(m_y-1)P,p_2+(m_x-1)P}^l\end{pmatrix} \tag{2.22}$$

  one of the P^2 polyphase components of the high resolution filter H^l (see eq. (1.10)).

By stacking all vectors and matrices coming from equation (2.19) for all $l = 1 : L$, we obtain the following model:

$$\begin{pmatrix}X_{LR}^1\\ \vdots \\ X_{LR}^L\end{pmatrix} = \begin{pmatrix}\mathcal{H}_{0,0}^1 & \dots & \mathcal{H}_{P-1,P-1}^1\\ \vdots & & \vdots \\ \mathcal{H}_{0,0}^L & \dots & \mathcal{H}_{P-1,P-1}^L\end{pmatrix}\begin{pmatrix}D_{0,0}\\ \vdots \\ D_{P-1,P-1}\end{pmatrix} + B_{LR} \tag{2.23}$$

The superresolution problem is now expressed like a multiple input multiple output problem. In multiple input systems, the inputs usually come from different sources, and are considered as independent from each other [5]. In our case, the inputs are the different subsampled components of the same source image and are therefore strongly correlated.

2.2 Limits of the subspace method

In this section, we show that, for subsampled images, the subspace method is not sufficient to determine the filters, but provide an identification up to a (P^2, P^2) mixing matrix.

Let us call $\mathbb{R}_X^{LR}$ the autocorrelation matrix of the L subsampled images X_{LR}^l.
If we apply the subspace method, we find that the eigenvectors, denoted G_i, associated to the $d_h = P^2(n_y+m_y-1)(n_x+m_x-1)$ greater eigenvalues of $\mathbb{R}_X^{LR}$ span the signal subspace, and that the eigenvectors associated to the $Ln_yn_x - d_h$ smaller eigenvalues of $\mathbb{R}_X^{LR}$ span the noise subspace.

The orthogonality condition between noise and signal subspaces is expressed by:

$$\underset{(1,Ln_yn_x)}{G_i^T} \quad \underset{(Ln_yn_x,\, s_ys_x)}{\mathcal{H}_{p_1,p_2}} = \underset{(1,\, s_ys_x)}{\mathbf{0}_{(1,s_ys_x)}} \tag{2.24}$$

where $i = 0 : Ln_yn_x - d_h - 1$, $\mathbf{0}_{(1,s_ys_x)}$ is a null vector of size $(1, s_ys_x)$, and $\mathcal{H}_{p_1,p_2}$ a block column of the filtering matrix in equation (2.23).

$$\mathcal{H}_{p_1,p_2} = \begin{pmatrix} \mathcal{H}_{p_1,p_2}^1 \\ \vdots \\ \mathcal{H}_{p_1,p_2}^L \end{pmatrix} \tag{2.25}$$

The structural lemma (see eq. (1.19)) provide an expression of the orthogonality condition in terms of the polyphase components of the filters instead of the columns of the filtering matrix:

$$\mathbb{H}_{p_1,p_2}^T \mathcal{G}_i = \mathbf{0}_{(1,s_ys_x)} \quad \text{where} \quad \mathbb{H}_{p_1,p_2} = \begin{pmatrix} H_{p_1,p_2}^1 \\ \vdots \\ H_{p_1,p_2}^L \end{pmatrix} \tag{2.26}$$

where $\mathcal{G}_i$ is a (Lm_ym_x, s_ys_x) filtering matrix defined from the eigenvectors G_i, and $\mathbb{H}_{p_1,p_2}$, for $p_1, p_2 = 0 : P-1$ is of size $(Lm_ym_x, 1)$.

By stacking the contributions of all the polyphase components of the filters, we obtain:

$$\mathbb{H}^T \mathcal{G}_i = \mathbf{0}_{(P^2,s_ys_x)} \quad \text{where} \quad \mathbb{H} = \begin{pmatrix} \mathbb{H}_{0,0} & \dots & \mathbb{H}_{P-1,P-1} \end{pmatrix} \tag{2.27}$$

The minimization of the quadratic form associated to the orthogonality condition provide a set of P^2 vectors, denoted $\mathbb{V}$. We can not distinguish these eigenvectors using only the orthogonality condition. Indeed, each column of $\mathbb{V}$ is in the null space of the quadratic form, therefore $\mathbb{H}$ is a combination of the P^2 columns of $\mathbb{V}$. We can identify the filters $\mathbb{H}$ only up to a reversible (P^2, P^2) mixing matrix denoted $\mathbb{M}_X$, such as:

$$\mathbb{H} = \mathbb{V}\mathbb{M}_X \tag{2.28}$$

Source separation methods have been used to estimate such a matrix [1, 5], but these methods usually state the assumption that the input signals are uncorrelated. This is not our case, as the inputs are the different subsampled components of the same source image.

2.3 Evaluation of the Mixing Matrix

The determination of the matrix $\mathbb{M}_X$ is, as we showed theoretically, impossible in the case where the mixed sources (here the polyphase components of an image) have the same distribution.
Despite this fact, we try to estimate the mixing matrix by introducing some prior knowledge on the statistics of the image or the filters. Indeed, natural images have a spectrum which is far from constant (as in the case of a white noise or a compressed signal). On the other hand, filters that are encountered in image processing are often very smooth with a single local (and global) maximum at the origin, whereas a multi-reflection filter, that affects wireless communications, can be irregular and display a multitude of local maxima. The subspace method was designed to deal with such irregular filters, with the counterpart that the sources are of different statistical nature, allowing an efficient separation of sources.

In this section we will use a continous notation, and the Fourier transform of a sampled signal at rate 1 will live in $[-1/2, 1/2]$ whereas the Fourier transform of a subsampled version at rate P, will live in $[-1/2P, 1/2P]$. The $\tilde{H}^l$ will refer to the estimated filters we are trying to define.

2.3.1 Imposing Regularity of the Filters

First, let us see what happens when some regularity is imposed to the filters. We do so by minimizing a certain regularity measure of the filters under the constraint that the integral of each filter is one[1].

Two principal choices have been proposed for the measure of filters regularity. The first one (which presents the advantage of a low computational cost) is the integral of the squared norm of the gradient (the $\mathbb{H}_1$ norm [10]). The other one is the integral of the gradient (the total variation norm [9]). The first choice may lead to smooth solutions and disadvantages the non continuous filters (such as motion blur). Nevertheless, we use this $\mathbb{H}_1$ criterion, for two reasons:

- We search for the best solution in a small-dimensional affine space (namely the vector space in which $\mathbb{M}_X$ lives intersected with the affine space represented by the constraint $\int H^l(x)dx = 1$). In such a case, the smoothing effect of the $\mathbb{H}_1$ norm compared to the TV norm could be ignored.

- The computational cost of such a minimization is much smaller than the TV one (see for example [2] for the numerical intricacy of TV minimization, although recent advances have been made [3] but are not, as is, applicable to our problem).

$$J_1(\tilde{H}^1, ..., \tilde{H}^L) = \sum_l \int \left\| \nabla \tilde{H}^l \right\|_2^2. \tag{2.29}$$

[1] This is a physical requirement for imaging filters. It may not be true if different images have been acquired under different illumination conditions. In this case, the mean of each image gives a very accurate estimation of the integral of the filter that generated it.

2.3.2 Imposing Similarity of the Double-Filtered Images

In the following we take advantage of the fact that we have multiple views of the same original scene to recover the filters (which implies the estimation of $\mathbb{M}_X$). Let's assume that we have two versions of the same image I_1 and I_2 formed after being filtered by F_1 and F_2, and that we have two candidates $\tilde{F_1}$ and $\tilde{F_2}$: we can check easily if these candidates are reasonable or not. Indeed filtering I_2 using $\tilde{F_1}$ should yield the same result as filtering I_1 using $\tilde{F_2}$.

Based on this simple observation, we define a functional which should be minimized by our computed filters:

$$J_2(\tilde{H}^1, ..., \tilde{H}^L) = \sum_{k,l=1}^{k,l=L} \left\| \tilde{H}^l * X^k - \tilde{H}^k * X^l \right\|_2^2 . \tag{2.30}$$

where X^k are the observed images and $\tilde{H}^k$ are the estimated filters.
Note that we don't have access to a fully sampled version of the X^k, thus we interpret the convolutions that occur in (2.30) as the product of the low frequencies of the filter $\tilde{H}$ with the Fourier transform of X, squaring the result and summing over the low-frequency domain.

We define[2]

$$\left\| \tilde{H}^l * X^k - \tilde{H}^k * X^l \right\|_2^2 = \int_{-\frac{1}{2P}}^{\frac{1}{2P}} \left| \hat{\tilde{H}}^l(u)\hat{X}^k(u) - \hat{\tilde{H}}^k(u)\hat{X}^l(u) \right|^2 du \tag{2.31}$$

This last functional could be the perfect criterion if no subsampling were present. Indeed, J_2 is null in a noise-free, well-sampled setting only if the filters are the real filters (after checking that J_2 is a positive definite quadratic form). Unfortunatly the subsampling that affects our images is expressed by :

$$\begin{aligned}
& \left| \hat{H}^l(u)\hat{X}^k(u) - \hat{H}^k(u)\hat{X}^l(u) \right|^2 \\
= & \left| \hat{H}^l(u) \sum_{n=0}^{P-1} \hat{X}^0(u+\frac{n}{P})\hat{H}^k(u+\frac{n}{P}) - \hat{H}^k(u) \sum_{n=0}^{P-1} \hat{X}^0(u+\frac{n}{P})\hat{H}^l(u+\frac{n}{P}) \right|^2 \\
= & \left| \sum_{n=1}^{P-1} \hat{X}^0(u+\frac{n}{P}) \left(\hat{H}^k(u)\hat{H}^l(u+\frac{n}{P}) - \hat{H}^l(u)\hat{H}^k(u+\frac{n}{P}) \right) \right|^2
\end{aligned} \tag{2.32}$$

for $u \in [-\frac{1}{2P}, \frac{1}{2P}]$,
where the H^k are the actual filters and X^0 is the original image.

Criterion J_2 being not null when applied to the actual filters prevents us from concluding that its minimum is obtained for those filters. Nevertheless, images have a strong low-frequency component. This means that the minimizing filters for J_2 must reduce as much as possible the terms of the form $|\hat{\tilde{H}}^k(u)\hat{X}^0(u) - \hat{\tilde{H}}^l(u)\hat{X}^0(u)|^2$, because these terms dominate the others (see [11] for a review of proposed statistical models of images). As the experiments will show it, the error introduced by the aliasing is

[2] We use a one dimensional notation to simplify the equations, we consider an infinite-size discrete signal subsampled at rate P. The hat denotes the time-discrete Fourier transform of a signal

neglictable and does not lead to a noticeable error in the recovery of the filters.
One can also say that the high frequency components of the filters are not taken into account. Although this point is correct, the filters, thanks to the subspace method, are constrained to live in a small-dimensional affine space, thus controlling the low frequency part of them is sufficient to yield a positive definite quadratic form on the subspace the filters live in.
In the next section we see how these two ideas can be applied to the disambiguation of the mixing matrix $\mathbb{M}_X$.

Chapter 3

Restoration

Once the filters are estimated, the recovery of the original image can take place. The recovered image D_{opt} must satisfy some straightforward conditions, namely :

- The image filtered by the estimated filters H^l and subsampled must be close to the observed images, which yields the first data-driven functional:

$$A(D_{opt}) = \sum_{l=1}^{L} \left\| S_P(H^l * D_{opt}) - X^l \right\|_2^2, \tag{3.1}$$

  S_P being the subsampling operator at rate P.

- Since the observed images are affected by noise and, most importantly, the filters we computed are estimates of the actual ones, a regularization functional must also be minimized:

$$R(D_{opt}) = \int \|\nabla D_{opt}\|_2^2 \tag{3.2}$$

These two criteria sum up to the minimization of a single functional given by:

$$J_3(D_{opt}) = A(D_{opt}) + \lambda R(D_{opt}), \tag{3.3}$$

Let us denote A_l the operator including the convolution by the filter H^l followed by the subsampling operator at rate P:

$$A_l(D) = S_P(H^l * D), \tag{3.4}$$

for $l = 1$ to L.

The minimum D_{opt} is obtained by solving the following system:

$$\left(\sum_{l=1}^{L} A_l^T \; A_l + \lambda \, C^T \; C \right) \; D_{opt} = \sum_{l=1}^{L} A_l^T \; X^l \tag{3.5}$$

where C denotes the gradient operator.
The system is solved in the frequency domain, as the operations of convolution and subsampling benefit from an easy and fast implementation.

First let us study the operator A_l: this operator provides the convolution of the input signal D with the filter H^l and subsamples the result at a rate P. In the frequency domain, this operation is equivalent to multiply the Discrete Fourier Transform (DFT) of the input signal, denoted $\widehat{D}$, by the DFT of the filter H^l, denoted $\widehat{H}^l$, and then to take into account the aliasing resulting from the subsampling :

$$\begin{aligned}\widehat{A_l D}(w) &= \widehat{D}(w)\,\widehat{H^l}(w) + \widehat{D}(w+N)\,\widehat{H^l}(w+N) + \cdots \\ &+ \widehat{D}(w+(P-1))\,\widehat{H^l}(w+(P-1)N)\end{aligned} \tag{3.6}$$

where the DFT of each low resolution signal are computed on N uniformly-spaced samples.

Now we focus on the operator A_l^T: this operator provides the upsampling of the input signal at a rate P, followed by the convolution of the result by the filter $(H^l)^H$, the conjugate transpose of H^l.

$$A_l^T(D) = (H^l)^T * S_P(D) \tag{3.7}$$

In the frequency domain, this operation is equivalent to duplicate P times the spectrum of D, weighted by $\frac{1}{P}$, then to mutiply the result by $(\widehat{H}^l)^H$:

$$\begin{pmatrix} \widehat{A_l^T D}(w) \\ \widehat{A_l^T D}(w+N) \\ \vdots \\ \widehat{A_l^T D}(w+(P-1)N) \end{pmatrix} = \frac{1}{P} \begin{pmatrix} \widehat{H^l}^*(w) \\ \widehat{H^l}^*(w+N) \\ \vdots \\ \widehat{H^l}^*(w+(P-1)N) \end{pmatrix} \widehat{D}(w) \tag{3.8}$$

where $w = 1:N$ and $\widehat{H^l}^*(w)$ is the conjugate of $\widehat{H^l}(w)$.

We deduce the formulation of the equation (3.5) in the fourier domain :

$$\left(\frac{1}{P}\sum_{l=0}^{L-1}\left((\widehat{\mathbf{H^l}})^H(w)\,\widehat{\mathbf{H^l}}(w)\right) + \lambda\,\widehat{\mathbf{C_2}}(w)\right)\widehat{\mathbf{D_{opt}}}(w) = \frac{1}{P}\sum_{l=0}^{L-1}(\widehat{\mathbf{H^l}})^H(w)\,\widehat{\mathbf{X}}_l^{LR}(w) \tag{3.9}$$

where

$$\widehat{\mathbf{H^l}}(w) = \left[\widehat{H^l}(w)\;\widehat{H^l}(w+N)\;\cdots\;\widehat{H^l}(w+(P-1)N)\right] \tag{3.10}$$

$$\widehat{\mathbf{D_{opt}}}(w) = \left[\widehat{D}_{opt}(w)\;\widehat{D}_{opt}(w+N)\;\cdots\;\widehat{D}_{opt}(w+(P-1)N)\right] \tag{3.11}$$

$$\widehat{\mathbf{C_2}}(w) = \left[|\widehat{C}(w)|^2\;|\widehat{C}(w+N)|^2\;\cdots\;|\widehat{C}(w+(P-1)N)|^2\right] \tag{3.12}$$

and $w = 1:N$.

Chapter 4

Applications

We want to estimate a deconvolved image, at a resolution increased by a factor $P = 2$, from a set of $L = 6$ low-resolution images of the same scene, filtered by 6 different unknown filters. This can be expressed as a 4 input 6 output system.

To evaluate the results with an objective criterion, the psnr (see eq. (4.1)), we have to simulate this case: we filter a known original image D with 6 known filters H, and then subsample the outputs by a factor $P = 2$ in each directions.

$$PSNR(D, D_{est}) = 10\ \log_{10} \frac{(\max(D) - \min(D))^2}{MSE(D, D_{est})} \tag{4.1}$$

where MSE is the mean squared error between the images D and D_{est}.

The original image D is $(576, 720)$, and the windowed area of study $(10, 10)$. The filters are $(6, 6)$ 2D-Gaussian centered at a random point with standard deviations: $0.7, 0.9, 1, 1.1, 1.3, 1.5$.
To recover the filters, we use a weighted sum of the two criteria on the filters, defined in eq. (2.29) and eq. (2.30) page 19: $\alpha J1 + (1 - \alpha)J2$. Note that J_1 and J_2 are normalized so their minimal eigenvalue is 1.
We obtain a psnr of 22.12 dB for $\alpha = 1$, and a psnr of 21.46 dB for $\alpha = 0$. The results are better when the two criteria are mixed, in our case for $\alpha = 0.04$, the filters are recovered with a psnr of 26.17 dB.

We present experimental results obtained with the observed output subsampled images and the filters estimated below.
Figure 4.1 shows 3 of the 6 output images, from the less blurred on the left, to the the more blurred on the right.
Figure 4.2 shows the restored image ($psnr = 26.78$ dB with $\lambda = 10^{-3}$ in eq. (3.3)) versus the original image.
To better display the results, we focus on a window area of the less blurred output image, and the related window area in the superresolved image, and display them at their exact size (figure 4.3). For comparison purposes, a bilinear interpolation of the output image area and the related window in the original image are also given.

Figure 4.1: 3 of the 6 output images: frame 1,3 and 6

Figure 4.2: upper : the original image, down : the restored image

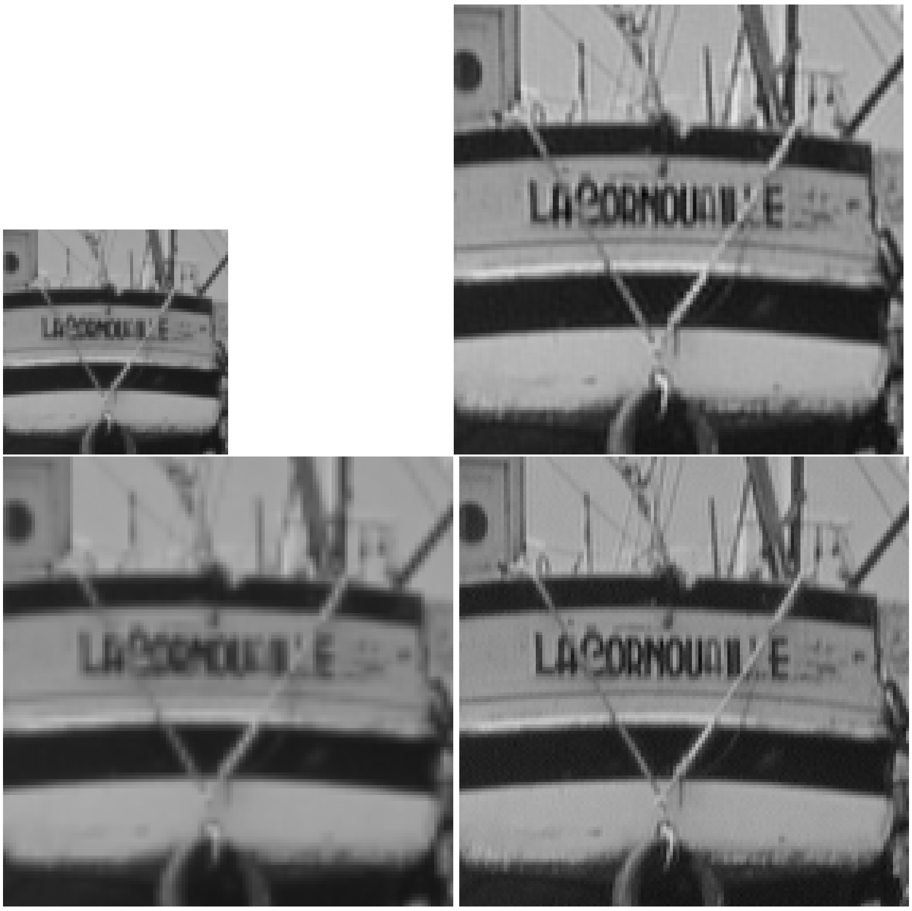

Figure 4.3: upper left : the 1st observed image; upper right : the restored image; down left : the bilinear interpolation; down right : the original image

Chapter 5

Conclusion

In this work we showed how the subspace method may be applied to image superresolution. We showed that this method is intrinsically ambiguous when presented with multiple sources which are, in fact, subsamples of the one same image. We showed how statistical properties of images can be used to disambiguate the problem and achieve a satisfactory recovery of the filters and of the original image. The advantage of using this method is that it can be applied to a wide range of filters without further assumption than their smoothness. In future work, one may want to apply other types of regularization to the image or the filters. The most promising lead is the TV regularization [3] which would be available as a usable technology very soon. The other possibility of improvement is the extension to the case where the made algebraic assumptions fail to be true, in such cases subspace method happens to be very unstable. We may apply the ideas presented here to stabilize the problem.

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Notations

Signals or images :

- X :
  - signal : stacks the L output signals of size N and is of size $(LN, 1)$:

$$X = \begin{pmatrix} X^1 \\ \vdots \\ X^L \end{pmatrix} \text{ where } X^l = \begin{pmatrix} x_0^l & \dots & x_{N-1}^l \end{pmatrix}^T \tag{1}$$

  - image : stacks the L observed images X^l, for $l = 1$ to L, more precisely, a vectorized formulation of a processing windowed area, of size $(N_y\ ,\ N_x)$, extracted from the observed images :

$$X^l = [x^l(N_y - 1, N_x - 1)\ x^l(N_y - 2, N_x - 1)\ \cdots\ x^l(0,0)]^T \tag{2}$$

- D :
  - signal : is the unknown original signal of size $(N + M - 1, 1)$
  - image : is a vectorized formulation of the related windowed area of the original image of size $(N_y + M_y - 1, N_x - M_x - 1)$,

$$D = [d(N_y + M_y - 2, N_x + M_x - 2) \cdots\ d(0,0)]^T \tag{3}$$

Subsampled signal or images :

- D_p : (signal) for $p = 0$ to $P - 1$, denotes a subsampled component of D

$$D_p = [d_p\ d_{p+P} \dots d_{p+(n+m-2)P}]^T \qquad (n + m - 1, 1) \tag{4}$$

- D_{p_1,p_2} : (images) is a vectorized form of

$$D_{p_1,p_2} = \begin{pmatrix} d_{p_1,p_2} & d_{p_1,p_2+P} & \cdots & d_{p_1,p_2+P(s_x-1)} \\ \vdots & \vdots & & \vdots \\ d_{p_1+P(s_y-1),0} & d_{p_1+P(s_y-1),P} & \cdots & d_{p_1+P(s_y-1),p_2+P(s_x-1)} \end{pmatrix} \tag{5}$$

for all $p_1, p_2 = 0 : P - 1$, where $s_y = n_y + m_y - 1$ and $s_x = n_x + m_x - 1$. Note that we use the same notation for the vectorized and the matrix forms of D_{p_1,p_2}

Filters:

- H : the filters coefficients

$$H = \begin{pmatrix} H^1 \\ \vdots \\ H^L \end{pmatrix} \tag{6}$$

- H^l :
  - 1D : is a filter of size $(M, 1)$:

$$H^l = [h_0^l \dots h_{M-1}^l]^T \tag{7}$$

  - 2D : is a filter of size (M_y, M_x)

$$H^l = \begin{pmatrix} h^l(0,0) & \dots & h^l(0, M_x - 1) \\ \vdots & & \vdots \\ h^l(M_y - 1, 0) & \dots & h^l(M_y - 1, M_x - 1) \end{pmatrix} \tag{8}$$

- H_p^l : (1D) one of the P subsampled components of the filter H^l

$$H_p^l = \left[\; h_p \; h_{p+P} \; \dots \; h_{p+(m-1)P} \;\right] \tag{9}$$

- H_{p_1,p_2}^l : (2D) one of the P^2 polyphase components of the filter H^l

$$H_{p_1,p_2}^l = \begin{pmatrix} h_{p_1,p_2}^l & \dots & h_{p_1,p_2+(m_x-1)P}^l \\ h_{p_1+P,p_2}^l & \dots & h_{p_1+P,p_2+(m_x-1)P}^l \\ \vdots & & \vdots \\ h_{p_1+(m_y-1)P,p_2}^l & \dots & h_{p_1+(m_y-1)P,p_2+(m_x-1)P}^l \end{pmatrix} \tag{10}$$

- $\mathbb{H}$: (2D) a formulation of H in terms of its polyphase components

$$\mathbb{H} = \begin{pmatrix} \mathbb{H}_{0,0} & \dots & \mathbb{H}_{P-1,P-1} \end{pmatrix} \tag{11}$$

  where

$$\mathbb{H}_{p_1,p_2} = \begin{pmatrix} H_{p_1,p_2}^1 \\ \vdots \\ H_{p_1,p_2}^L \end{pmatrix} \tag{12}$$

Filtering matrices:

- $\mathcal{H}$:
  - signal : stacks the L Toeplitz matrices $\mathcal{H}^l$:

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}^1 \\ \vdots \\ \mathcal{H}^L \end{pmatrix} \qquad (LN, N + M - 1) \tag{13}$$

where $\mathcal{H}^l$, for $l = 1$ to L, is the filtering matrix associated to the filter H^l :

$$\mathcal{H}^l = \begin{pmatrix} h_0^l & \dots & h_{M-1}^l & & 0 \\ & \ddots & & \ddots & \\ 0 & & h_0^l & \dots & h_{M-1}^l \end{pmatrix} \qquad (N, N+M-1) \tag{14}$$

– image : stacks the L block-Toeplitz filtering matrices $\mathcal{H}^l$ associated with each filters H^l :

$$\mathcal{H}^l = \begin{pmatrix} \mathcal{H}_0^l & \cdots & \mathcal{H}_{M_x-1}^l & & 0 \\ & \ddots & & \ddots & \\ 0 & & \mathcal{H}_0^l & \cdots & \mathcal{H}_{M_x-1}^l \end{pmatrix} \tag{15}$$

where $\mathcal{H}_j^l$ is a Toeplitz matrix of size $(N_y\ ,\ N_y + M_y - 1)$ associated to the j^{th} column of H^l :

$$\mathcal{H}_j^l = \begin{pmatrix} h^l(0,j) & \dots & h^l(M_y-1,j) & & 0 \\ & \ddots & & \ddots & \\ 0 & & h^l(0,j) & \dots & h^l(M_y-1,j) \end{pmatrix} \tag{16}$$

$\mathcal{H}^l$ contains N_x rows of blocks and $N_x + M_x - 1$ columns of blocks of size $(N_y\ ,\ N_y + M_y - 1)$. $\mathcal{H}$ is of size $(LN_yN_x\ ,\ (N_y + M_y - 1)(N_x + M_x - 1))$

- $\mathcal{H}_p^l$: (signal) the filtering matrix associated to H_p^l

$$\mathcal{H}_p^l = \begin{pmatrix} h_p^l & h_{p+P}^l & \dots & h_{p+(m-1)P}^l & & 0 \\ & \ddots & & & \ddots & \\ 0 & & h_p^l & h_{p+P}^l & \dots & h_{p+(m-1)P}^l \end{pmatrix} \qquad (n, n+m-1) \tag{17}$$

- $\mathcal{H}_{p_1,p_2}^l$: (image) is the block-Toeplitz matrix of size (n_yn_x, s_ys_x) associated to the filter H_{p_1,p_2}^l
- $\mathcal{H}_{LR}$: the filtering matrix of the MIMO system
  – signal :

$$\begin{pmatrix} \mathcal{H}_0^1 & \dots & \mathcal{H}_{P-1}^1 \\ \vdots & & \vdots \\ \mathcal{H}_0^L & \dots & \mathcal{H}_{P-1}^L \end{pmatrix} \tag{18}$$

  – image :

$$\begin{pmatrix} \mathcal{H}_{0,0}^1 & \dots & \mathcal{H}_{P-1,P-1}^1 \\ \vdots & & \vdots \\ \mathcal{H}_{0,0}^L & \dots & \mathcal{H}_{P-1,P-1}^L \end{pmatrix} \tag{19}$$

- $\mathcal{H}_{p_1,p_2}$: (image) a block column of the filtering matrix $\mathcal{H}_{LR}$

$$\mathcal{H}_{p_1,p_2} = \begin{pmatrix} \mathcal{H}_{p_1,p_2}^1 \\ \vdots \\ \mathcal{H}_{p_1,p_2}^L \end{pmatrix} \tag{20}$$

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